

Algebra Diagnostic

Texas A & M University

Simplifying Expressions

1. Rationalize the denominator: $\frac{9}{4 + \sqrt{7}}$.

a. $\frac{36 - \sqrt{7}}{9}$

b. $4 + \sqrt{7}$

c. $\frac{36 + 9\sqrt{7}}{23}$

d. $4 - \sqrt{7}$

e. $\frac{9 - \sqrt{7}}{4}$

2. Given $g(x) = 3x^2 - x$ and $h(x) = \sqrt{x+1}$, find the value of $(g \circ h)(3)$.

a. 10

b. 48

c. 5

d. 14

e. 9

3. Find the sum or difference as indicated, and write your answer in simplified form.

$$\frac{x+h-3}{x+h} - \frac{x+3}{x}$$

a. $\frac{2xh+3h}{x^2+xh}$

b. $\frac{2x^2+2xh+3h}{x^2+xh}$

c. $\frac{-6x-3h}{x^2+xh}$

d. $\frac{-x^2-2x-xh-2h-3}{x+h}$

e. $\frac{h-6}{h}$

4. Factor and reduce to simplest form: $\frac{6x^2 - 7xy - 3y^2}{2x^2 - 5xy + 3y^2}$.
- $\frac{3x + y}{x + y}$
 - $\frac{3x + y}{x - y}$
 - $\frac{3x - y}{x + y}$
 - $\frac{3x - y}{x - y}$
 - $\frac{3x - y}{x - 3y}$

Exponents

5. Simplify the following completely: $\frac{(x^{-1}y^{\frac{1}{3}})^{-\frac{1}{2}}}{x^{\frac{1}{3}}y^{-\frac{1}{2}}}$.
- $x^{-\frac{5}{6}}y^{\frac{2}{3}}$
 - $x^{\frac{1}{6}}y^{\frac{1}{3}}$
 - $x^{\frac{5}{6}}y^{-\frac{2}{3}}$
 - $x^{\frac{1}{6}}y^{-\frac{2}{3}}$
 - $x^{-\frac{5}{6}}y^{\frac{1}{3}}$
6. Perform the indicated operation and simplify your answer: $(ab\sqrt[3]{2ab^2})(2b\sqrt[3]{4ab})$.
- $4a^{\frac{5}{3}}b^3$
 - $4a^{\frac{5}{3}}b^2$
 - $4a^{\frac{4}{3}}b^3$
 - $4a^{\frac{4}{3}}b^2$
 - $4a^{\frac{5}{3}}b^{\frac{13}{3}}$

Solving Equations

7. Solve the following equation: $2(x - 2)^2 + 3(x - 2) + 1 = 0$. The sum of the roots is:
- $\frac{7}{2}$
 - $\frac{11}{2}$
 - $\frac{9}{2}$
 - $-\frac{3}{2}$
 - $\frac{5}{2}$

8. Solve for a : $\sqrt{a-2} + 8 = a$. The sum of all solutions is:
- a. -11
 - b. 17
 - c. 11
 - d. -6
 - e. 6
9. Solve for x : $9^{3x+1} = 27^{3-2x}$.
- a. $x = \frac{2}{5}$
 - b. $x = 0$
 - c. $x = \frac{12}{7}$
 - d. $x = \frac{5}{2}$
 - e. $x = \frac{7}{12}$
10. Find the point (x, y) which satisfies both equations. What is the value of $x + y$?
- $$\begin{cases} 3x - 5y = 6 \\ 2x - 3y = 1 \end{cases}$$
- a. -9
 - b. -13
 - c. -17
 - d. -22
 - e. -4

Word Problems

11. Two investments are made totaling \$8,000. In one year these investments yield \$538 in simple interest. Part of the \$8,000 is invested at $6\frac{1}{2}\%$, and the rest at $7\frac{3}{4}\%$. How much more money is invested at $6\frac{1}{2}\%$?
- a. \$1,440.00
 - b. \$6,849.12
 - c. \$5,120.00
 - d. \$575.44
 - e. \$6,560.00

12. Given the linear equation $ax + by = c$, where $a, b, c > 0$, if x decreases by 4 units, what is the corresponding change in y ?
- a. y decreases by $\frac{4a}{b}$
 - b. y decreases by $\frac{a}{b}$
 - c. y increases by $\frac{4b}{a}$
 - d. y increases by $\frac{4a}{b}$
 - e. y increases by $\frac{a}{b}$
13. Line A passes through the points $(k + 3, k + 2)$ and $(5, -4)$. Find the value of k , if line A has a slope of 0.
- a. $k = -4$
 - b. $k = -6$
 - c. $k = 2$
 - d. $k = 3$
 - e. $k = -7$

Quadratics

14. Find the vertex of the function $f(x) = (3p)x^2 - (24p)x + (5p)$.
- a. $(3, -40p)$
 - b. $(6, -31p)$
 - c. $(4, -43p)$
 - d. $(1, -16p)$
 - e. $(0, 5p)$
15. Beginning with $f(x) = x^2$, which of the following shows $f(x)$ shifted to the left 1 unit, reflected about the x axis, and then shifted down 4 units?
- a. $g(x) = -(x + 1)^2 - 4$
 - b. $g(x) = -(x - 4)^2 + 1$
 - c. $g(x) = -(x - 1)^2 - 4$
 - d. $g(x) = -(x - 4)^2 - 1$
 - e. $g(x) = (1 - x)^2 - 4$

Solving Inequalities

16. Solve for x : $\frac{2x+4}{x-3} \leq 4$.

a. $(-\infty, -2] \cup (3, \infty)$

b. $(-\infty, 3) \cup [8, \infty)$

c. $[-2, 3)$

d. $(3, 8]$

e. $[-4, 3)$

Finding domains

17. Find the domain of $f(x) = \frac{\sqrt{3x^2 - 75}}{x^2 - 4x - 21}$.

a. $(-\infty, -5] \cup [5, \infty)$

b. $(-\infty, -3) \cup [5, \infty)$

c. $(-\infty, -3) \cup [5, 7) \cup (7, \infty)$

d. $\mathbb{R}, x \geq 5, x \neq 7$

e. $(-\infty, -5] \cup [5, 7) \cup (7, \infty)$

18. Find the domain of the function $f(x) = \begin{cases} \frac{2x-6}{x+5}, & x < -1 \\ \frac{x-3}{x+3}, & x \geq -1 \end{cases}$

a. $\mathbb{R}, x \neq -3$

b. $\mathbb{R}, x \neq -5$

c. $\mathbb{R}, x \neq -3, -5$

d. $\mathbb{R}, x \neq -3, -5, -1$

e. $\mathbb{R}, x \neq -1$

Finding Intercepts and Asymptotes

19. Find the x -intercept(s) of the function $f(x) = \frac{3x^2 + 13x - 10}{2x^2 + 13x + 15}$, if any exist.

a. $x = -\frac{3}{2}, -5$

b. $x = \frac{2}{3}, -5$

c. $x = -\frac{3}{2}$

d. $x = \frac{2}{3}$

e. no x -intercept

20. Find the vertical asymptote(s) of the function $f(x) = \frac{3x^2 + 13x - 10}{2x^2 + 13x + 15}$, if any exist.
- a. $x = -\frac{3}{2}$
 - b. $x = -\frac{3}{2}, x = -5$
 - c. $x = -5$
 - d. $x = \frac{2}{3}$
 - e. no vertical asymptote

Logarithms

21. Solve for x : $\log(2x - 14) + \log(x - 2) = 2$.
- a. $x = 3, x = 12$
 - b. $x = 3$
 - c. $x = 12$
 - d. $x = -3$
 - e. $x = -3, x = 12$

22. Simplify the following expression:

$$\frac{\ln(e^2) - \log_3(81)}{\ln(e^{x^2}) - \log(10)}$$

- a. $\frac{-1}{2x - 1}$
- b. $\frac{-8}{x^2 - 1}$
- c. $\frac{-1}{x^2 - 1}$
- d. $\frac{e^2 - 4}{x^2 - 1}$
- e. $\frac{-2}{x^2 - 1}$

23. Given $\log_a 3 = 5$, $\log_a 7 = -2$, and $\log_a 11 = 4$, find $\log_a \frac{77}{3a^2}$.
- a. -1
 - b. -5
 - c. $-\frac{8}{10}$
 - d. $-\frac{8}{15}$
 - e. -3

24. Solve for x : $4 \ln(6x) = 20$.

a. $x = \frac{1}{6}e^5$

b. $x = e^{\frac{5}{6}}$

c. $x = 5 - \ln 6$

d. $x = e^5 - \ln 6$

e. $x = e^5 - 6$

Absolute Value

25. Write as a piece-wise function without absolute value signs: $f(x) = |24 - 6x|$.

a. $f(x) = \begin{cases} 24 - 6x, & x \geq 0 \\ 6x - 24, & x < 0 \end{cases}$

b. $f(x) = \begin{cases} 24 - 6x, & x \geq 4 \\ 6x - 24, & x < 4 \end{cases}$

c. $f(x) = \begin{cases} 4 - x, & x \geq 1 \\ x - 4, & x < 1 \end{cases}$

d. $f(x) = \begin{cases} 24 - 6x, & x \leq 4 \\ 6x - 24, & x > 4 \end{cases}$

e. $f(x) = \begin{cases} 4 - x, & x < 1 \\ x - 4, & x \geq 1 \end{cases}$

Answer Key

- | | | | | |
|-------------|--------------|--------------|--------------|--------------|
| 1. <i>d</i> | 6. <i>a</i> | 11. <i>c</i> | 16. <i>b</i> | 21. <i>c</i> |
| 2. <i>a</i> | 7. <i>e</i> | 12. <i>d</i> | 17. <i>e</i> | 22. <i>e</i> |
| 3. <i>c</i> | 8. <i>c</i> | 13. <i>b</i> | 18. <i>b</i> | 23. <i>b</i> |
| 4. <i>b</i> | 9. <i>e</i> | 14. <i>c</i> | 19. <i>d</i> | 24. <i>a</i> |
| 5. <i>b</i> | 10. <i>d</i> | 15. <i>a</i> | 20. <i>a</i> | 25. <i>d</i> |